Supplementary Material to Article:
“A Natural-Scene Gradient Distribution Prior and its Application in Light-Microscopy Image Processing”

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1 Supplementary Figures

Figure 1: CPU time for our model (Eq. 7 in main text) and model gradient evaluations, compared with the hyper-Laplacian model. Timings using Matlab on an Apple MacBook Pro (early 2011).
Figure 2: The gradient CDF (bottom row) is sensitive to image transformations (original image is from: beyondthehumaneye.blogspot.de). For the blurred image (Gaussian blur, $\sigma = 3$), the frequency of small gradients is increased. For the noisy image (10% Gaussian noise), the frequency of large gradients is increased. For the zoomed (SR) image (upsampling factor 9), the frequency of small gradients is increased. For the bilateral filter ($w = 5$, $\sigma_s = 3$, $\sigma_c = 0.1$) and the guided filter ($r = 10$, $\epsilon = 0.01$), the frequency of small gradients is increased.
(a) Marginal gradient distributions in log scale of all training images (one curve per image). Color (coded by color) and other parameters fixed at indicates scaled density of curves.

(b) Our model with changing parameter $a_2$ (coded by color) and other parameters fixed at their best fit: $b_2 = 5.4$, $c_2 = -0.266$.

(c) Hyper-Laplacian model with changing parameter $b_1$ (color) and other parameters fixed at their best fit.

Figure 3: Sensitivity analysis of our model compared with the hyper-Laplacian model. Parameter $a_2$ varies from $1 \times 10^{-4}$ to $4 \times 10^{-4}$ with step size $2 \times 10^{-5}$. For the hyper-Laplacian model, $b_1$ varies from 0.5 to 0.6 with step size 0.01.
Figure 4: (a) Average distributions of Gaussian curvature, mean curvature, and Laplace operator response across all training images of our natural-scene image dataset. (b) The naturalness factors computed from the gradient and the Laplacian distributions are highly correlated.
2 Supplementary Tables

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Table 1: Summary of Poisson solvers. The FFT and Wavelet-based solvers are implemented in our software. The total number of pixels in the image is \((nm)\), with \(n\) and \(m\) the numbers of pixels along each dimension.

\(^8\)dense Cholesky decomposition
\(^9\)sparse Cholesky decomposition
3 Supplementary Text

3.1 Proof of Lemma VII.1

Proof. Let $v = |\nabla U|^2$ and $W = a_{pr} + \frac{b_{pr} - v}{(b_{pr} + v)^2} = 0$. Then we have quadratic equation $a_{pr}(b_{pr} + v)^2 + b_{pr} - v = 0$.

If $a_{pr}b_{pr} \geq \frac{1}{8}$, then $W \geq 0$.

If $a_{pr}b_{pr} < \frac{1}{8}$, then $v_L = \frac{1 - 2a_{pr}b_{pr} - \sqrt{1 - 8a_{pr}b_{pr}}}{2a_{pr}}$ and $v_U = \frac{1 - 2a_{pr}b_{pr} + \sqrt{1 - 8a_{pr}b_{pr}}}{2a_{pr}}$ such that

\[
\begin{cases} 
W < 0 & \text{when } v_L \leq v \leq v_U \\
W > 0 & \text{otherwise}
\end{cases}
\] (1)

\[\square\]
3.2 Zooming / Super Resolution

Zooming or super-resolution (SR) is the process of up-sampling an image (or a part of an image) onto a larger grid of pixels. Increasing the number of pixels in the image while keeping the field of view the same hence increases the image resolution, albeit not the optical resolution. The interesting question is then how to interpolate the image information onto the finer pixel grid where no information is available on the coarse input grid. We show here how the same algorithm as used in the main text for deconvolution can also be used for zooming.

The only change from the deconvolution model is that we use an up-sampled $U$ (i.e., $U$ has more pixels than $I$) and a known Gaussian kernel $K(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-|x|^2/2\sigma^2}$. We hence do not need to use alternating minimization, because the kernel is known in this application. An example is shown in Suppl. Fig. 5.

It is instructive to compare the resulting zoomed image with an image of the same sample acquired by a true super-resolution microscopy technique (here: PALM microscopy, photo-activated localization microscopy).

While zooming with the present algorithm renders the image crisper (due to the deconvolution kernel) and better resolved (due to the finer pixel grid), it does not actually improve the optical resolution of the microscope. This can nicely be observed when two filaments cross. In the zoomed image there is a gap at the crossing point, whereas the PALM microscopy image properly resolves both filaments crossing.
Figure 5: Zooming using the GDP. Panel (b) shows the zoomed version (up-sampling factor 4) of the fluorescently labeled microtubules in (a) as computed using the present method. Panel (c) shows a real super-resolution PALM image of the same scene for comparison. ((a)&(c) from: EPFL Collection of Reference Datasets, bigwww.epfl.ch/smlm/datasets/index.html?p=real-hd)
3.3 GDP in image windows

Figure 6: Gradient distributions in local image regions are invariant, provided the regions are large enough for stable histogram estimation. (a) An example microscopy image with three different regions unpainted in different color. (b) The gradient histograms in these three regions of corresponding color. (c) A transmission electron micrograph (ssTEM) image of the \textit{Drosophila} first instar larva ventral nerve cord (VNC) with a resolution of \(4 \times 4 \times 50\) nm/pixel [1]. (d) The gradient distributions within manually segmented mitochondria and synapses (all regions pooled) are almost identical.

Since the GDP is stable with image contents, it is also valid on sub-images and image regions. As shown in Suppl. Fig. 6(a,b), the gradient distribution is insensitive to the position, size, and shape of the patch. This is confirmed for an electron-microscopy image in Suppl. Fig. 6(c,d). The image shows a transmission electron micrograph (ssTEM) of the \textit{Drosophila} first instar larva ventral nerve cord (VNC) from [1]. The gradient distributions within manually segmented mitochondria and synapses (all regions pooled) are
almost identical, as shown in Suppl. Fig. 6(d). This suggests that the GDP can be straightforwardly extended to multi-region methods.

Clearly, the GDP loses its validity when applied to small image patches that contain little or no internal structure. As the size of a local window decreases, we transition from a macroscopic view (entropy) to a microscopic view (pixel histogram). Since the GDP is a macroscopic quantity, it is only valid for large-enough image patches where the gradient histogram can be estimated with statistical significance. But how large is large enough? Unfortunately, there is no sharp transition. To quantitatively see this, we define the naturalness map for a local window of edge length $w$ as:

$$N_w(x, y) = \int \int p(\vec{G}) \log \left( \frac{p(\vec{G})}{p_{\text{pr}}} \right) \, d\vec{G} \, d\vec{G},$$

where $\hat{x} \in [x - w, x + w], \hat{y} \in [y - w, y + w]$. This quantifies the distance (KL-divergence) between the GDP and the gradient distribution in each local window. Computed for every image patch, this provides a map of how the image naturalness varies across space. Two examples are shown in Suppl. Fig. 7. The average and median values of $N_w$ across all patches are plotted in Suppl. Fig. 7(e,j) when the window size decreases from 60 to 8 pixel edge length $w$. These plots show how the gradient distribution gradually diverges from the macroscopic GDP as the window size decreases. It seems that this behavior of prior invalidation is independent of image contents, as shown in Suppl. Fig. 8.

Figure 7: Decreasing the local window size $w$, the gradient distribution prior increasingly differs from the empirical distribution within the windows. The original image is shown followed by naturalness maps computed in increasingly smaller moving window sizes $w$. The panels (e,j) show how both the mean and the median distance $N_w$ between the GDP and all local windows gradually diverge with decreasing window size $w$. 

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Figure 8: Three examples suggesting that the behavior of $N_w$ with patch size is independent of image contents.
References