Variable-metric randomized black-box optimization over the cone of positive definite matrices

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Abstract

Many tasks in science, engineering, and control can be formulated as a continuous optimization problem over the cone of positive (semi-)definite matrices. A specific problem class where efficient algorithms exists both in theory and practice are semi-definite programs (SDP). Semi-definite programs consists of linear objective functions over the intersection of the cone of positive (semi-)definite matrices with an affine space. Recently, a number of randomized algorithms have been proposed for this problem class that come with theoretical convergence guarantees under certain assumptions. One example is Kleiner et al.'s Random Conic Pursuit algorithm, a randomized approximation algorithm for SDP's [1] of the form:

$$\min_{\mathbf{X}\in\mathbb{S}^n} f(\mathbf{X}) \qquad \mathbf{X} \succeq 0\,, \tag{1}$$

where f is a convex function. Dabbene and co-workers [2] introduced a randomized cutting plane method with geometric convergence for problems of the form:

$$\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x}) \qquad \text{s.t.}\mathbf{A}_0 + x_1\mathbf{A}_1 + \ldots + x_n\mathbf{A}_n \preceq 0\,, \tag{2}$$

where $\mathbf{A}_i \in \mathbb{S}^n$ are symmetric matrices that define the constraints on $\mathbf{x} = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ and f is a linear function. In this contribution we will revisit these algorithms and discuss the key proof ideas and convergence rates. We will then show how we can combine certain ideas from these algorithms with variable-metric stochastic search methods [3] such as Gaussian Adaptation (GaA) [4] and CMA-ES [5]. Firstly, we show how the use of fast-mixing Markov Chain Monte Carlo methods and analytic boundary oracles considered in [2] can be used in CMA-ES or GaA to tackle real-valued black-box function optimization with semi-definite constraints. Secondly, we propose a reinterpretation of CMA-ES and GaA as an adaptive random walk over the cone of positive (semi-)definite matrices. This allows to heuristically tackle black-box problems over positive definite matrices. Provable convergence rates cannot be established for the presented schemes at the moment. Nonetheless, we believe that the proposed concepts may be of some interest to the community because it establishes a link between state-of-the-art randomized search heuristics and state-of-the-art randomized convex optimization algorithms.

References

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