Spatiotemporal Modeling and Simulation

09 :: Advection-Diffusion
Topic

Advection-Diffusion

Program

Governing equation for advection-diffusion
Advection-diffusion simulation using particles
Stability of the discretization
Remeshing
Interpolation schemes
Self-test questions
Learning goals

Be able to simulate advection-diffusion systems with PSE and RW

Be able to define compressible and incompressible advection mathematically

Know the stability condition and the Lagrangian CFL condition

Know why remeshing is needed and be able to implement it

Be able to explain the particle cloud and assignment function of interpolation

Know the first three members of the hierarchy
0th order: Nearest Grid Point (NGP)

Order: 0
Support: $1^d$
Interpolated field: Piecewise constant (stepwise)
1st order: Cloud In Cell (CIC)

Particle cloud

Assignment function

Order: 1
Support: $2^d$
Interpolated field: Piecewise linear (continuous values)
2nd order: Triangular Shaped Cloud (TSC)

Particle cloud

Assignment function

Order: 2
Support: $3^d$
Interpolated field: Continuous value and gradient
3rd order: Monaghan’s $M'_4$ function

$$M'_4(s) = \begin{cases} 1 - \frac{1}{2}(5s^2 - 3s^3) & , 0 \leq s < 1 \\ \frac{1}{2}(2 - s)^2(1 - s) & , 1 \leq s \leq 2 \\ 0 & , s > 2 \end{cases}$$

Order: 3
Support: $4^d$
Interpolated field: Continuous value, gradient, and second derivatives
### Summary

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Order</th>
<th>Number of points</th>
<th>Cloud shape</th>
<th>Assignment function shape</th>
<th>Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGP</td>
<td>0</td>
<td>$1^d$</td>
<td>$\delta$</td>
<td>$\Pi$</td>
<td>Stepwise</td>
</tr>
<tr>
<td>CIC</td>
<td>1</td>
<td>$2^d$</td>
<td>$\Pi$</td>
<td>$\Lambda = \Pi \ast \Pi$</td>
<td>Continuous piecewise linear</td>
</tr>
<tr>
<td>TSC</td>
<td>2</td>
<td>$3^d$</td>
<td>$\Lambda$</td>
<td>$\Pi \ast \Pi \ast \Pi$</td>
<td>Continuous value and first derivative</td>
</tr>
<tr>
<td>PQS</td>
<td>3</td>
<td>$4^d$</td>
<td>$\Lambda \ast \Pi$</td>
<td>$\Pi \ast \Pi \ast \Pi \ast \Pi$</td>
<td>Continuous value first and second derivative</td>
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