Topic: Partial differential equations

Lecture: Spatiotemporal Modeling and Simulation

Duration: 10 minutes

The following questions are for you to check whether you have understood the contents of the lecture. Please reply to the questions alone (for yourself) and by writing on this sheet of paper. As soon as you and your neighbor are both done, you might want to discuss your answers. Your answers will not be collected or graded. It’s a pure self-test for your convenience. If you are not able to easily answer the questions or if you have doubts regarding the correctness of your replies, please take a few minutes to read in the book or the slides in order to revisit the corresponding points.

1. Recall the definition of partial differential equations. What is the defining distinction from ODEs? Describe it in your own words in one sentence! (K1)

2. Classify the following PDEs with respect to their order and type: (K3)
   i) \( \nabla \cdot \mathbf{u} = 0 \) (incompressible continuity equation)
   
   ii) \( \frac{i\hbar \partial u}{\partial t} = -\frac{\hbar^2}{2m} \Delta u + V(x) \) (Schrödinger equation)
   
   iii) \( \rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \nu \Delta u \) (Navier-Stokes equation)

3. You have hard about the notion of “ill-posed” problems. Explain to a friend, who has never heard about this, what it means and why it should be avoided in scientific simulation models! (K2)
4. What side conditions (boundary and initial conditions) are required to solve a parabolic PDE? Try to imagine what parabolicity physically means and give the answer without looking it up in your notes! (K2)

5. Consider the Tricomic PDE:

\[ y \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0. \]

It’s a linear, homogeneous PDE of 2nd order with varying coefficients. What is the sub-class (hyperbolic, parabolic, elliptic) of this equation? Be careful: it depends on the value of \( y \). Give the sub-classes for all cases and the ranges of \( y \) in which they are valid! (K3)

6. (Optional) Besides the methods of Fourier transform and separation of variables, there is also the method of characteristics, which is, however, limited to hyperbolic equations. Although the method is not as intuitive as the other two, it has a nice physical interpretation. Revisit the example of the wave equation. What is the physical meaning of the characteristics of the wave equation? Explain in a few sentences! (K2)