Exercise 13
Release: 20.01.2020
Due: 27.01.2020

Question 1: Stability
Consider the Linear Differential Equation system
\[ \dot{x} = \begin{pmatrix} -1001 & 999 \\ 999 & -1001 \end{pmatrix} x \]

a) we want to apply the explicit Euler and the implicit Trapezoidal procedure to this system. How big must the step size be for the numerical solution for stable qualitative behaviors, when starting with the following initial conditions?
   i) \( x(0) = (-1, 1)^T \)
   ii) \( x(0) = (1, 1)^T \)
   iii) \( x(0) = (2, 0)^T \)

Question 2: Stiff Differential Equations
Consider the IVP, 
\[ y'(t) = -15y(t) \]
with the initial condition \( y(0) = 1 \) and \( t \geq 0 \).

a) Find the exact analytical solution to the problem.

b) Use explicit Euler scheme to solve the problem with step size \( h = \frac{1}{4} \) and \( h = \frac{1}{8} \)

c) Use the implicit Trapezoidal scheme to solve the problem and plot a graph comparing explicit Euler \( (h = \frac{1}{4}, \frac{1}{8}) \), implicit Trapezoidal \( (h = \frac{1}{8}) \), and the analytical solution.

Question 3: Classification of PDEs
Classify the following PDEs with respect to their order and type (parabolic, hyperbolic and elliptic)

\[ \frac{\partial u}{\partial t} = D \Delta u \]
\[ \Delta u = 0 \]
\[ u_{tt} - c^2 u_{xx} = 0 \]
\[ u_{xx} + xu_{yy} = 0 \]