Exercise 14
Release: 27.01.2020
Due: 03.02.2020

Question 1: Solution of PDEs
Consider the PDE for advection equation \( u_t + cu_x = 0 \). Assuming that, we are only allowed to Fourier transform along \( x \), i.e.

\[
\hat{u}(\xi, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x, t) e^{-i\xi x} dx
\]

a) Formulate the analytical solution for \( u(x, t) \) given the initial data for \( \hat{u}(\xi, 0) \).
b) Repeat the same analysis for diffusion equation \( u_t = Du_{xx} \).

HINT: Take the fourier transform of the respective equations

Question 2: Semi-analytical solution
Given the PDE for heat conduction,

\[
u_t = u_{xx} - \cos(2\pi x), \quad x \in \mathbb{R}\]

with boundary conditions \( u(t, 0) = u(t, 1) = 0 \) and Initial conditions \( u(0, x) = 0 \).

Approximate the solution by applying the Method of Lines with the implicit Euler procedure for time integration. Formulating the method and then calculate an example with \( h = 0.01, t_f = 10 \).

Question 3: Programming Task
Consider the poisson equation

\[
\nabla^2 u(x, y) = 1 \quad (x, y) \in \Omega := (-1, 1) \times (-1, 1) \\
u = 0 \text{ at } \partial \Omega
\]

a) Discretize the problem with 5-point FD stencil.
b) Solve the linear system and plot the solution.